

Seismic model-based inversion using Matlab

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1. Introduction

This chapter presents a workflow to invert post-stack seismic reflection data into acoustic impedance through the sequential use of several Matlab® codes. Acoustic impedance is defined as the product of density and seismic velocity and, as such, it is an intrinsic physical property of rocks. This physical property is closely related to variables that are of fundamental importance in the context of hydrocarbon reservoir characterization, such as lithology, porosity and, in some cases, water or oil saturation.

The basic premise of the vast majority of seismic inversion methods is the local validity of the 1-D convolutional model. Recursive methods were developed first in the late 70s (e.g. Lavergne and Willm, 1977; Lindseth, 1979) while sparse-spike methods were developed in the 80s. The later consists of techniques that use an additional premise that the reflections occur as sparsely distributed spikes within a layered Earth (e.g. Russell, 1988). Nowadays both methods are still widely jointly used even in sophisticated commercial seismic processing packages. Two well known methods that fall in this category are the L_1 -norm sparse-spike inversion (e.g. Sacchi and Ulrych, 1996) and the maximum likelihood inversion (e.g. Hampson and Russell, 1988). When the sparse-spike inversion is constrained by a low-frequency model derived from acoustic impedance well logs or geologic models, it is commonly referred to as model-based inversion (Russel, 1988).

The idea of the proposed workflow is to apply a L_1 -norm sparse-spike inversion algorithm in the time domain, followed by a recursive inversion performed in the frequency domain. A low-frequency impedance model estimated at well-logs is incorporated as constraints during the recursive process. While it is clear that a similar inversion methodology can readily be applied using commercial packages, there is no consistent workflow designed for this type of application in low-cost scientific platforms such as Matlab. Therefore, the processing and visualization tools presented here are potentially useful especially for academic users of seismic data aiming at reservoir characterization.

2. Seismic-Well Tie

Before applying seismic inversion, an accurate depth-to-time conversion must be performed in order to make the vertical scale of the well log *AI* data match the vertical scale of the seismic data so as to allow spatial correlation. This conversion is carried out by using the sonic log and the initial two-way travel time for the first log sample that provides the

highest correlation coefficient between synthetic and observed trace. This is commonly known as seismic-well tie (e.g White and Hu, 1998).

Synthetic traces are calculated using the convolutional model given by Equation 1. This requires knowledge of the wavelet that represents the seismic pulse. By writing Equation 1 as a linear system and solving for $w(t)$, a deterministic wavelet extraction is conducted (Broadhead, 2008).

3. Model-Based Seismic Inversion

The fundamental premises behind all seismic inversion methods in the context of this work are: (i) the Earth can be represented locally by a stack of plane and parallel layers with constant physical properties; (ii) the seismic trace $s(t)$ can be represented by the convolution of the reflectivity coefficient series $r(t)$ with a band-limited wavelet $w(t)$ and the addition of a random noise $n(t)$:

$$s(t) = r(t) * w(t) + n(t). \quad (1)$$

For zero incident angles, $r(t)$ is directly related to the contrast in the acoustic impedance (AI) of superposed layers through a simple equation that, after some algebraic manipulations and mathematical approximations, leads to the expression

$$AI_M = AI_1 \exp\left(2 \sum_{j=2}^M r_j\right), \quad (2)$$

which is the equation used in practice for recursive inversion with the aim of transform reflectivities into impedances. AI_1 is the known acoustic impedance in the top layer and AI_M is that of the M^{th} layer. r_j is the reflection coefficient of the j^{th} layer. This approximation is valid for most of the practical cases where $r_j \leq |0.3|$ (e.g. Berteussen & Ursin, 1983).

The low-frequency AI model is obtained by estimating the AI values over the entire seismic volume through ordinary kriging of the AI values at the wells. The AI values at the wells are obtained by simple multiplication of the measured density values and the inverted sonic logs (i.e. interval transit time). For a properly usage of the recursive inversion, the seismic traces should be deconvolved into reflectivity series as suggested by Equation 2. To accomplish that, one has to apply a constrained sparse-spike optimization procedure that minimizes the objective function

$$J(\mathbf{r}) = \alpha \sum_{j=1}^M |r_j| + \frac{1}{2} \left\| \frac{1}{\sigma} (\mathbf{s} - \mathbf{W}\mathbf{r}) \right\|^2 \quad (3)$$

using, for example, a conjugate-gradient algorithm. The first term of Equation 3 minimizes the L_1 -norm of the reflectivities where α controls the sparsity of the solution. The second term minimizes the difference between the synthetic seismic traces ($\mathbf{W}\mathbf{r}$) and the observed traces ($\mathbf{s}$). $\mathbf{W}$ is a wavelet coefficient matrix and σ is the standard deviation of the seismic data noise.

After estimating $\mathbf{r}$ from the seismic amplitudes, then $\mathbf{r}$ is inverted into AI according to the following sequential steps (Ferguson and Margrave, 1996):

- (1) compute the linear trend of a spatial correspondent AI vector and subtract it, obtaining a residual AI_{res} vector;
- (2) compute the Fourier spectra of AI_{res} ;
- (3) apply Equation (2) to the reflectivity series, obtaining a relative AI_{rel} vector;
- (4) compute the Fourier spectra of AI_{rel} ;
- (5) determine a scalar α to match the mean power of AI_{rel} and AI_{res} ;
- (6) multiply the spectra of AI_{rel} by α ;
- (7) low-pass filter AI_{res} and add to the result of step (6);
- (8) inverse Fourier transform the result of step (7); and
- (9) add the low-frequency trend from step (1) to the result of step (8).

For the particular dataset used in this work, the wells are sparsely distributed through the oil field (Figure 1). Thus, the low-frequency trend of step (1) was extracted from the spatial correspondent AI trace estimated by kriging. A low cut-off for coupling the low frequency trend and a high cut-off is defined by finding where the energy content of the original seismic traces approaches to zero in the amplitude spectrum. This characterizes the band-limited nature of the seismic data. The basic workflow is presented in Figure 2.

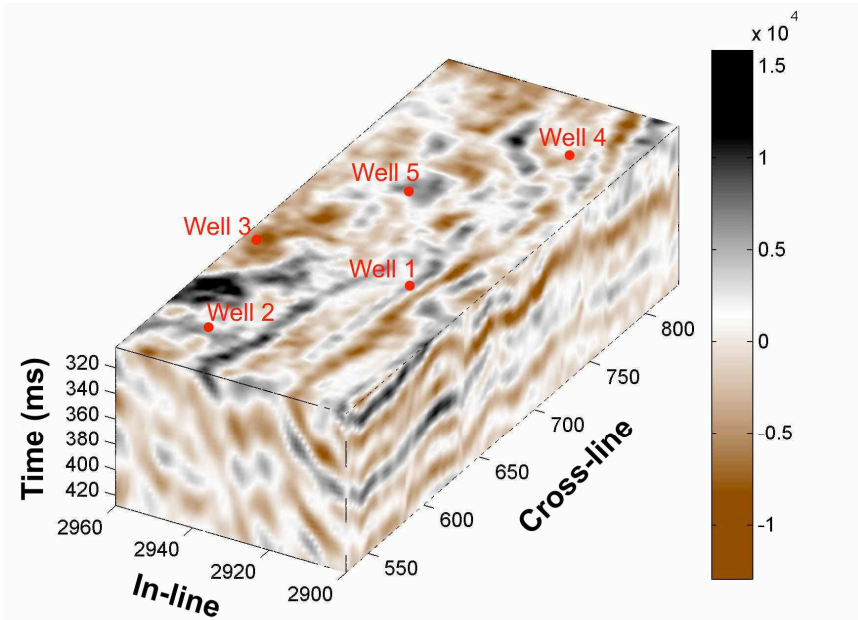


Fig. 1. 3D seismic data and spatial location of wells. Size of 3D matrix is $301 \times 61 \times 375$. In-lines and cross-lines are spaced of about 13 and 27 m, respectively. Time interval is equal to 4 ms.

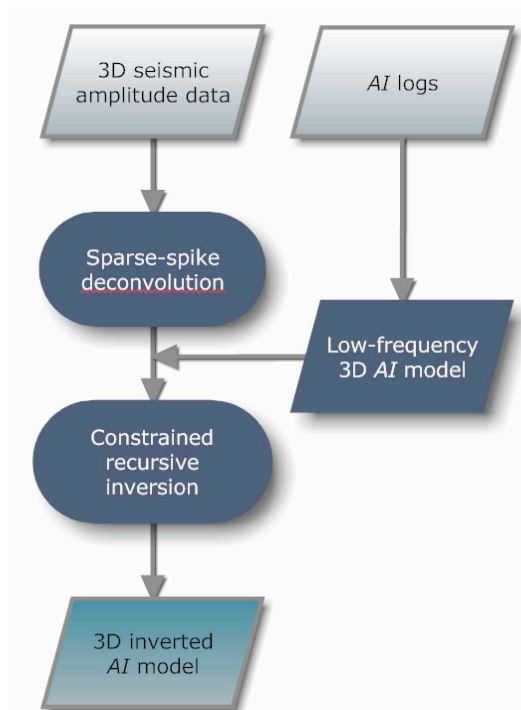


Fig. 2. Flowchart of the proposed inversion methodology. Each AI log is obtained by multiplying density log and inverted sonic log.

4. Matlab Algorithms

Seismic-well tie is conducted using functions from Seislab 3.01¹. The core functions are *l_depth2time* and *s_wavextra*. The former computes two-way travel time by inverting and integrating sonic log starting from a given depth and a given time. Because at the beginning of the process we do not know the correct depth/time pair to be used, a range of values must be tested. The later performs deterministic wavelet extraction, as long as a reflectivity series and an observed seismic trace around the well are provided. The reflectivity series is obtained by rearranging Equation 2 and solving for each $r(t)$. *s_wavextra* also outputs a synthetic trace and the correlation coefficient with respect to the observed trace. The best depth/time pair is that for which this correlation coefficient is higher.

Sparse-spike inversion is applied by using the function *sparse_decon* from SeismicLab package². This function performs L_1 regularization with Iterative Reweighted Least Squares (Sacchi, 1997). In this step, a reflectivity series is obtained from an observed seismic trace and a wavelet for each vertical column of seismic data.

¹ <http://www.mathworks.fr/matlabcentral/fileexchange/15674>

² <http://www-geo.phys.ualberta.ca/saig/SeismicLab/index.html>

To generate low-frequency models from known AI values at the wells, ordinary kriging is applied using mGstat³ function *krig*. mGstat is a very flexible package for geoestatistical analysis and also provides interfaces to GSTAT, VISIM and SGeMS.

In the last step, recursive inversion is employed in order to map $AI(t)$ from $r(t)$. The *blint* function from CREWES⁴ group is used. This function calculates the summation that appears in Equation 2 in the frequency domain. Domain conversion is applied by using Matlab® native functions *fft* and *ifft* with few adaptations. Low and high frequency cut-offs must be provided and the integration filter rolls off as a smooth gaussian. This is not a problem because the seismic data is inherently of band-limited nature, so the task of the interpreter is to find these cut-offs.

To incorporate the low-frequency AI model, firstly log trends are subtracted from the AI logs. These log trends correspond to the lowest frequencies of the AI spectrum and they are simply fitted polynomials whose coefficients are calculated by using native Matlab® *polyfit* function. Its value for a given time is obtained by *polyval*, which is also a native Matlab® function. Having removed the trends, the logs are converted to the frequency domain and a low-pass filters are applied to them. Finally, the results are merged with the band-limited outputs from *blint* by using *mergetrcs* function. After merging, the output log is converted back to time domain and the log trends are restored.

Visualizations can be performed using functions *s_wplot* and *s_cplot* for 2D seismic data, *s_volume_browser* for 3D seismic data and *l_plot* for well log data. These are powerful functions found in SeisLab.

5. Application Example

An example of depth-to-time conversion for Well 2 can be visualized in Figure 3, where it is shown the acoustic impedance log, the estimated reflectivity, the synthetic traces and the observed traces. Seismic-well ties were conducted by adjusting five traces around each well and retaining the local means. A global mean was calculated and used for inversion of the traces away from the wells.

Figure 4 shows the spectral content of the reflectivity, the wavelet, the synthetic traces and the observed traces. The spectral content of the other four wells is similar and low and high cut-offs were defined as 5 Hz and 60 Hz respectively.

In this case, the hydrocarbon reservoir top and base is estimated from well log markers allowing the definition of a minimum and maximum time values, thus establishing vertical boundaries for the seismic 3D grid. Lateral boundaries is defined so as to embrace wells that were previously found to have some hydrocarbon content. The 3D AI inverted model is shown in Figure 5. The average correlation coefficient of a synthetic seismic model calculated from this inverted model and the observed seismic data is equal to 0.95.

³ <http://mgstat.sourceforge.net/>

⁴ <http://www.crewes.org/>

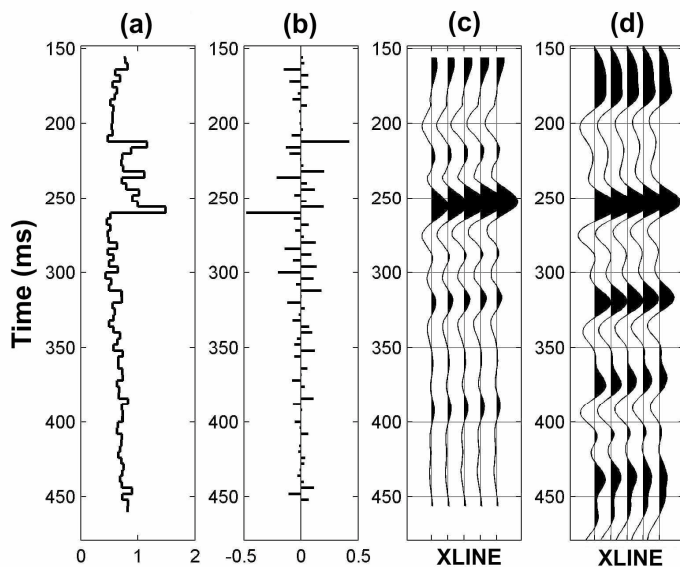


Fig. 3. Example of seismic-well tie for Well 2. (a) Impedance log converted to two-way time and resampled to the interval of 4 ms (values in $\text{m/s} \times \text{g/cm}^3 \times 10^4$). (b) Reflectivity. (c) Synthetic traces. (d) Observed traces.

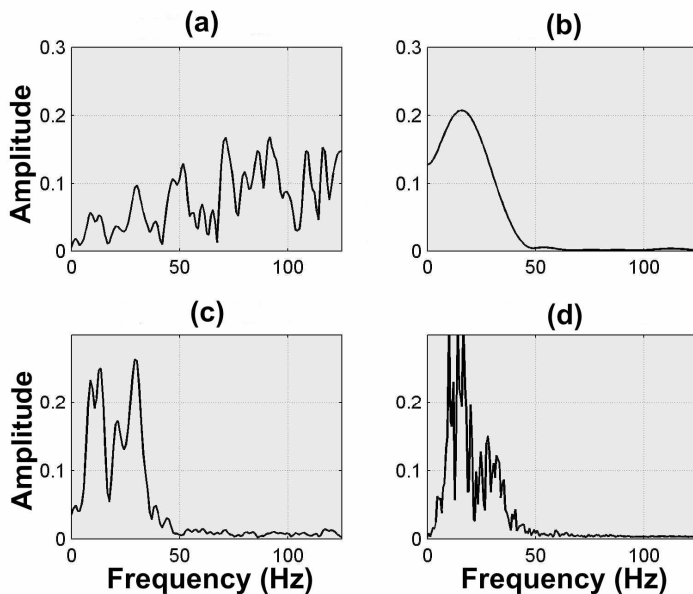


Fig. 4. Normalized Amplitude spectrum of (a) reflectivity; (b) wavelet; (c) synthetic traces; and (d) observed traces near Well 2.

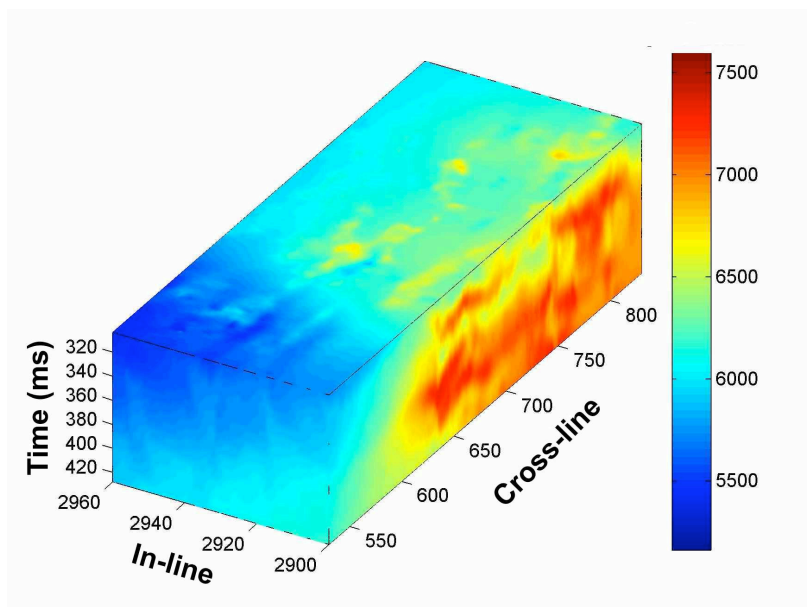


Fig. 5. AI obtained through the proposed inversion methodology. Color bar is in $\text{m/s} \times \text{g/cm}^3$.

6. Conclusions and recommendations

A simple methodology for mapping acoustic impedance and effective porosity from 3D seismic amplitude data using Matlab® was presented. This methodology can be used for a quick evaluation of reservoir properties, especially when powerful commercial programs are not available. An example with real data was also presented, showing that consistent 3D acoustic impedance models can be obtained if well-logs and 3D seismic data are available. A further improvement would be to obtain the low-frequency model by taking into account stratigraphic horizons, i.e., trend surfaces extracted from seismic data. This can be performed through universal kriging, or kriging with a trend. In Matlab®, universal kriging can be applied, for example, from mGstat toolbox. The use of stratigraphic horizons would allow the creation of low-frequency models that are spatially more consistent with the geological layering of a given reservoir area. Hopefully a Graphical User Interface will be developed in the near future integrating all this functions building a complete framework for performing seismic model-based inversion.

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8. References

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